Charmonium spectra at finite temperature from QCD sum rules with the maximum entropy method

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Charmonia spectral functions at finite temperature are studied using QCD sum rules in combination with the maximum entropy method. This approach enables us to directly obtain the spectral function from the sum rules, without having to introduce any specific assumption about its functional form. As a result, it is found that while J/ψ and η_c manifest themselves as significant peaks in the spectral function below the deconfinement temperature T_c, they quickly dissolve into the continuum and almost completely disappear at temperatures between 1.0 T_c and 1.1 T_c.

Since QCD was established to be the theory of strong interactions, charmonium has often been used as a suitable probe of its dynamics, owing to the fact that in this system both perturbative and nonperturbative aspects of QCD play equally important roles [1]. The behavior of charmonia in a hot or dense medium has also attracted much interest, as it was suggested some time ago, that in the color-deconfined medium with a temperature above T_c charmonia will dissolve due to the color Debye screening, and thus serve as a signal for the formation of quark-gluon plasma [2].

Testing these early suggestions from first principles of QCD has become feasible only recently, as new developments in lattice QCD have made it possible to access the charmonium spectral functions with the help of the maximum entropy method (MEM) [3–6]. These studies found that the lowest charmonium states (J/ψ and η_c) survive up to temperatures as high as $\sim 1.5\ T_c$ or higher.

Besides lattice QCD, the method of QCD sum rules [7] provides another tool for investigating the properties of hadrons at finite temperature [8, 9]. Using this approach various charmonium channels were studied recently [10, 11], and evidence for a considerable change of the spectral functions just above T_c was found. To specify the nature of this change is the major goal of this study. For this task we employ MEM, which is applicable to QCD sum rules [12] and has the advantage that one does not have to introduce any strong assumption about the functional form of the spectral function, such as the "pole + continuum" ansatz, which is often used in QCD sum rule studies.

Let us first recapitulate what sort of information QCD sum rules can provide on the charmonium spectral function at finite temperature [8, 9]. One considers the time-ordered correlator at finite temperature

$$\Pi^{\mathrm{J}}(q) = i \int d^4x e^{iqx} \langle T[j^{\mathrm{J}}(x)j^{\mathrm{J}}(0)] \rangle_T, \qquad (1)$$

where $j^{\mathrm{J}}(x)$ stands for $\bar{c}\gamma_\mu c(x)$ and $\bar{c}\gamma_5 c(x)$ in the vector (V) and pseudoscalar (PS) channel, respectively. The expectation value $\langle \mathcal{O} \rangle_T$ is defined as $\langle \mathcal{O} \rangle_T \equiv \mathrm{Tr}(e^{-H/T}\mathcal{O})/\mathrm{Tr}(e^{-H/T})$. Throughout this work, we will set the spatial momentum of the charmonium system relative to the thermal medium to be $\mathbf{0}$; thus, $q^\mu = (\omega, \mathbf{0})$. In this circumstance, there is only one independent component in the correlator of the vector channel. In what follows, we will use the dimensionless functions $\tilde{\Pi}^{\mathrm{V}}(q^2) \equiv \Pi_\mu^{\mu,\mathrm{V}}(q)/(-3q^2)$ and $\tilde{\Pi}^{\mathrm{PS}}(q^2) \equiv \Pi^{\mathrm{PS}}(q)/q^2$ for the analysis.

Going to the deep Euclidean region $q^2 \equiv -Q^2 \ll 0$, one can calculate the correlation functions using the operator product expansion (OPE), giving an expansion in local operators O_n with increasing mass dimension n: $\tilde{\Pi}^{\mathrm{J}}(q^2) = \sum_n C_n^{\mathrm{J}}(q^2)\langle O_n \rangle_T$. As was first discussed in [9], as long as the temperature T lies below the separation scale of the OPE, which is of the order of ~ 1 GeV, all the temperature effects can be included into the expectation values of the local operators $\langle O_n \rangle_T$, while the Wilson coefficients $C_n^{\mathrm{J}}(q^2)$ are independent of T. Furthermore, to improve the convergence of the OPE and suppressing the influence of high energy states onto the sum rule, we apply the Borel transform to the correlator, leading to the final result of the OPE for $\nu \equiv 4m_c^2/M^2$, M being the Borel mass:

$$\begin{aligned}\mathcal{M}^{\mathrm{J}}(\nu) =&\ e^{-\nu}A^{\mathrm{J}}(\nu)[1 + \alpha_s(\nu)a^{\mathrm{J}}(\nu) + b^{\mathrm{J}}(\nu)\phi_b(T) \\ &+ c^{\mathrm{J}}(\nu)\phi_c(T) + d^{\mathrm{J}}(\nu)\phi_d(T)].\end{aligned} \qquad (2)$$

The first two terms in Eq.(2) are the leading order perturbative term and its first order α_s correction. The third and fourth terms contain the scalar and twist-2 gluon condensates of mass dimension 4: $\phi_b(T) = \frac{4\pi^2}{9(4m_c^2)^2}G_0$ and $\phi_c(T) = \frac{4\pi^2}{3(4m_c^2)^2}G_2$, where $G_0 = \langle \frac{\alpha_s}{\pi}G_{\mu\nu}^a G^{a\mu\nu} \rangle_T$ and G_2 is defined as $\langle \frac{\alpha_s}{\pi}G^{a\mu\sigma}G_\sigma^{a\nu} \rangle_T = (u^\mu u^\nu - \frac{1}{4}g^{\mu\nu})G_2$, u^μ being the four velocity of the medium. For the detailed expressions of the Wilson coefficients of these terms, see

[11]. To evaluate the possible influence of higher order contributions, we include one more term, which is proportional to the scalar gluon condensate of dimension 6, $\phi_d(T) = \frac{1}{(4m_c^2)^3}\langle g^3 f^{abc} G_\mu^{a\nu} G_\nu^{b\lambda} G_\lambda^{c\mu}\rangle_T$. The Wilson coefficient of this term can be found in [13].

The correlator can also be expressed by a dispersion relation, in terms of the spectral function $\rho^{\rm J}(\omega)$ of the channel specified by the operator $j^{\rm J}(x)$. After the Borel transform one obtains

$$\mathcal{M}^{\rm J}(\nu) = \int_0^\infty dx^2 e^{-x^2\nu} \rho^{\rm J}(2m_c x). \qquad (3)$$

Equating Eqs.(2) and (3) then gives the final form of the sum rules. In the vector channel, an additional constant term contributes to Eq.(2), which originates from a pole at $\omega = 0$ in $\rho^{\rm V}(\omega)$ [8]. As this so-called scattering term considerably complicates the analysis, we eliminate it by taking the derivative of Eqs.(2) and (3) with respect to ν and analyze only the resulting derivative sum rule in this channel. For a discussion on the validity of this procedure in the heavy quark sum rules, see [14].

The usual strategy of analyzing QCD sum rules is to make some reasonable assumptions on the functional form of the spectral function, and then extract information on the lowest lying peak from Eqs.(2) and (3). This method, however, has several shortcomings. First of all, the widely used "pole + continuum" ansatz, which certainly works well at $T = 0$, may not be appropriate at temperatures above T_c, where the lowest lying state is expected to be modified and eventually melt into the c-$\bar{c}$ continuum, which could become the dominant contribution. Furthermore, it is not always possible to unambiguously fit a specific ansatz to the OPE results, because of the occurrence of equally valid solutions. Such a situation arose in [10, 11], where it was not possible to determine a unique solution for the used parametrization of the spectral function. To handle these problems, we propose to use MEM, which allows us to extract the spectral function from Eqs.(2) and (3) without prejudice on its functional form. Moreover, it can be proven that this method provides a unique solution for the spectral function [15].

Let us now briefly summarize the basic ideas of MEM, which helps us to carry out the task of inverting the integral of Eq.(3). This is, however, an ill-posed problem as we have only access to $\mathcal{M}^{\rm J}(\nu)$ in the region of ν where the OPE shows sufficient convergence and, furthermore, have only information on $\mathcal{M}^{\rm J}(\nu)$ with limited precision due to the uncertainties of the values of m_c, α_s and the various condensates. Nonetheless, using Bayesian probability theory, MEM makes it possible to at least obtain the most probable form of the spectral function $\rho(\omega)$. To do this, one defines a probability $P[\rho|\mathcal{MI}]$ for ρ to have a specific form given $\mathcal{M}$ and additional information $\mathcal{I}$ on ρ such as positivity and asymptotic values. Using Bayes' theorem, this probability can be rewritten as

$P[\rho|\mathcal{MI}] \propto P[\mathcal{M}|\rho\mathcal{I}]P[\rho|\mathcal{I}] = e^{-L+\alpha S}$, where e^{-L} is the likelihood function, used in standard χ^2 methods. $e^{\alpha S}$ stands for the prior probability, given by the parameter α and the Shannon-Jaynes entropy,

$$S = \int_0^\infty d\omega \Big[\rho(\omega) - m(\omega) - \rho(\omega)\log\Big(\frac{\rho(\omega)}{m(\omega)}\Big)\Big]. \qquad (4)$$

$m(\omega)$ is called the default model and can be used to incorporate prior knowledge on the asymptotic values of $\rho(\omega)$ into the analysis. Determining now the most probable $\rho(\omega)$ corresponds to the numerical problem of finding the maximum of the functional $-L + \alpha S$, for which we use the Bryan algorithm [16]. As a last step, the spectral function $\rho_\alpha(\omega)$ maximizing $-L + \alpha S$ for a fixed value of α is averaged using $\int d\alpha\rho_\alpha(\omega)P[\alpha|\mathcal{MI}]$, which gives the final output of the MEM analysis. For more details see [15], while specific issues concerning the application of this method to QCD sum rules are discussed in [12].

Next, we describe how the temperature dependencies of the gluonic condensates are determined. For the scalar and twist-2 gluon condensates with mass dimension 4, we follow the approach proposed in [10, 11], where, in the quenched approximation, the energy momentum tensor, expressed using gluonic operators, was matched with the corresponding quantity written down in form of the energy density ϵ and the pressure p, leading to $G_0 = G_0^{\rm vac} - \frac{8}{11}\big[\epsilon(T) - 3p(T)\big]$ and $G_2 = -\frac{\alpha_s(T)}{\pi}\big[\epsilon(T) + p(T)\big]$ for the scalar and twist-2 gluon condensates. The functions $\epsilon(T)$, $p(T)$ and $\alpha_s(T)$ were then extracted from quenched lattice QCD data [17, 18]. We will in this study use the same numerical values for the T dependent part of G_0 and G_2 as in [11]. As is shown there, both G_0 and G_2 exhibit a sudden decrease in the vicinity of T_c.

It was suggested in previous studies that the OPE could break down at temperatures above T_c as higher dimensional operators may become non-negligible [11]. To investigate this possibility, we include the scalar gluonic condensate with dimension 6, $\langle g^3 f^{abc} G_\mu^{a\nu} G_\nu^{b\lambda} G_\lambda^{c\mu}\rangle$, about which much less is known. To our knowledge, at $T = 0$, there exists only an estimate based on the dilute instanton gas model, giving $\langle g^3 f^{abc} G_\mu^{a\nu} G_\nu^{b\lambda} G_\lambda^{c\mu}\rangle = \frac{48\pi^2}{5\rho_c^2}\langle\frac{\alpha_s}{\pi} G^2\rangle$, where ρ_c is a representative value for the instanton radius, for which we use the established value of 0.33 fm [19]. Assuming that the relation above also holds at finite temperature, and taking into account the reduction of ρ_c above T_c [20], we, however, found that the dimension 6 term does not influence the behavior of the spectral function much in the temperature region investigated in this Letter. Therefore, we conclude that even though the relation obtained from the dilute instanton gas model can only be considered to be a crude estimate, as long as it gives the correct order of magnitude, the contribution of the dimension 6 condensate is small and does not alter the results obtained in this study.
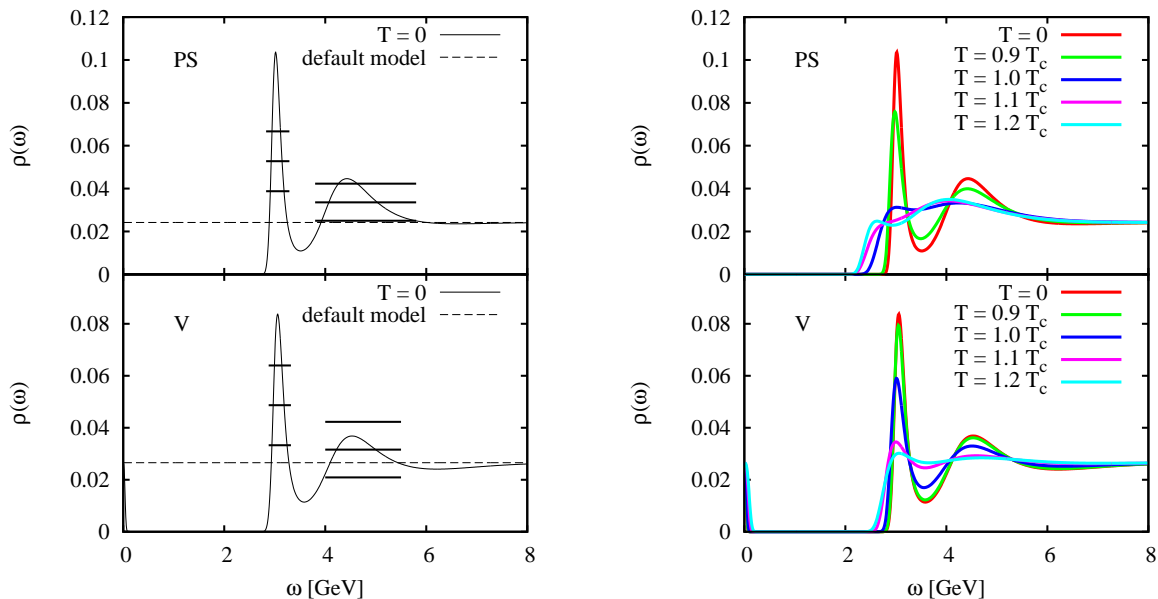
FIG. 1: On the left side, the spectral functions in the pseudoscalar (upper plot) and vector (lower plot) channel at $T = 0$, with the errors of their averaged values are shown. The horizontal extent and position of the error bars indicates the region over which the average is taken. The dashed lines show the default model used in the MEM analysis. On the right side, the corresponding spectral functions are given at temperatures around T_c.

Let us now turn to the MEM analysis of Eqs.(2) and (3). First, we investigate the spectral function at $T = 0$ in the vector and pseudoscalar channel. To determine the upper boundary of the region of ν to be analyzed, we employ the criterion that the dimension 6 term should be smaller than 20% of the whole OPE expression of Eq.(2), which gives $\nu_{\max}^{V} = 8.03$ in the vector and $\nu_{\max}^{PS} = 7.29$ in the pseudoscalar channel. We keep these values fixed when going to finite temperature. In fact, in the temperature region around T_c, the relative contribution of the dimension 6 term at $\nu_{\max}^{V,PS}$ is even smaller, namely, around 10% or less. The lower boundary of ν is chosen to be $\nu_{\min}^{V,PS} = 0.78$, corresponding to a Borel mass of $M = 3.0$ GeV. We have checked that the obtained spectral functions do not depend much on this choice. For the value of the charm quark mass m_c, we use a recent estimate giving $\overline{m}_c(m_c) = 1.277 \pm 0.026$ GeV [21], for α_s we employ the newest world average $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [22], while for the vacuum gluon condensate G_0^{vac} the standard value $G_0^{\mathrm{vac}} = 0.012 \pm 0.0036$ GeV^4 [7, 23] is applied. For the default model $m(\omega)$, we use a constant matched to the perturbative value of the spectral function at high energy, as was done in similar studies using lattice QCD [3].

The resulting spectral functions are given on the left side of Fig. 1. We observe in both channels a clear ground state peak, corresponding to η_c and J/ψ. The spectral functions also exhibit a second peak, which is, however, not statistically significant. These second peaks most likely reflect the existence of several excited states, which the MEM analysis is not able to resolve, quite similar to

the situation encountered in lattice studies. Furthermore, it is seen that the spectral function of the vector channel approaches the default model in the region close to $\omega = 0$, which, however, should not be confused with a contribution of the scattering term. This behavior is an artifact caused by our usage of the derivative sum rule in this channel and should thus not be considered to be a physical effect. Numerically, the peak representing η_c lies at 3.02 GeV, while the one standing for J/ψ is found at 3.06 GeV. Thus, we see that the ground state in both channels reproduces the experimental value with a precision of the order of 50 MeV. In the vector channel, the residue can be related to the electronic width of the corresponding resonance. We can obtain this residue from Fig. 1 simply by integrating the spectral function in the region of the peak, which gives 0.162 GeV^2, which is in good agreement with the experimental value of 0.173 GeV^2. On the other hand, we observe that the hyperfine splitting between η_c and J/ψ is underestimated. All these findings are in qualitative agreement with the results obtained in the conventional analysis of the charmonium sum rules [13].

Next, we increase the temperature according to Eq.(2). The resulting spectral functions are shown on the right side of Fig. 1 at temperatures between $0.9\,T_c$ and $1.2\,T_c$. It is seen in the figure that the behavior of the spectral functions changes abruptly in the vicinity of T_c. First, both ground state peaks experience a shift to lower energies of the order of 50 MeV, before dissolving quickly into the continuum above the critical temperature. Investigating the spectral functions in more detail, one ob-

serves that η_c disappears already at $T = 1.0\ T_c$, while J/ψ survives a bit longer, but also appears to be melted to a large degree at $T = 1.1\ T_c$. This sudden qualitative change of the spectral function mainly originates from the changes of the third and fourth terms in Eq.(2), which can be traced back to the rapid adjustment of the thermodynamic quantities $\epsilon(T)$ and $p(T)$ around T_c. It is reassuring to note that our results are consistent with the findings of [10, 11] in the sense that both observe a negative energy shift of the peaks around T_c. In these earlier works, it was, however, not possible to discuss the possible melting of the peaks because a relativistic Breit-Wigner form for the spectral function was assumed at all investigated temperatures.

For obtaining firm conclusions, one has to test the reliability of the MEM procedure at finite temperature, where systematic effects decrease the reproducibility and resolution of the spectral function obtained from MEM. In lattice studies, this reduced reliability is primarily caused by the reduction of the data points in the imaginary time correlator, due to periodicity and the reduction of the maximal time extent. In the case of QCD sum rules, this problem does not exist, as Eq.(2) is given as a continuous function at any temperature and therefore the same number of data points can be used. Nevertheless, the reliability of the MEM technique is still reduced at finite temperature due to the uncertainties of the thermodynamic functions $\epsilon(T)$ and $p(T)$, whose contribution grows with temperature and therefore increases the error of Eq.(2). In order to confirm that the change of the spectral function in Fig. 1 is not an artifact, we reanalyze Eq.(2) at $T = 0$, but use the errors of $T \neq 0$ in the analysis. The results are then compared to the ones given in Fig. 1, to investigate the net temperature effect on the spectral function. We find from this analysis that while the height of the peaks of the spectral functions at $T = 0$ is indeed reduced because of the increased error, this effect is much smaller than the actual reduction of the peaks around T_c, seen on the right side of Fig. 1. We therefore conclude that the disappearance of the peaks observed in Fig. 1 is a physical effect and is not induced by an artifact of the MEM analysis.

In summary, we have extracted the spectral functions of the pseudoscalar and vector channel at both zero and finite temperature using a combined analysis of QCD sum rules and MEM. At $T = 0$, the MEM technique is able to clearly resolve the lowest energy peaks, corresponding to the η_c and J/ψ resonances. The positions of both peaks agree with the experimental values with a precision of about 50 MeV. At finite temperature, we find that η_c and J/ψ melt quickly after the temperature is raised above the deconfinement temperature T_c, caused by the sudden change of the dimension-4, scalar and twist-2 gluon condensates in this temperature region. We have checked that this effect is not an artifact of the systematics of the MEM analysis. These results quantitatively disagree with the earlier findings of lattice studies which suggest that both η_c and J/ψ can survive at temperatures of up to $1.5\ T_c$ or higher. It, however, has to be mentioned that our results are in fact consistent with the latest lattice results [24], finding the peaks of η_c and J/ψ to be largely distorted between $0.73\ T_c$ and $1.46\ T_c$. It remains to be seen whether or not the two methods will converge to compatible conclusions when more accurate analyses will become available in the future.

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